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GROWTH OF DISTURBANCES IN A SUPERSONIC BOUNDARY LAYER

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UDC 532.526

The onset of turbulence in supersonic flows has stimulated investigations of the stability of compressible boundary layers. The first theoretical studies of this problem were reported by Lees, Lin, and Dunn (see Lin [1]). Attempts to verify the theory experimentally have been undertaken [2, 3], but the experiments were performed with natural disturbances, whose wave spectra were not controllable. Consequently, although spatially growing disturbances were successfully observed in [3], the comparison with the theoretical results was of a qualitative nature. The outcome in [2], on the other hand, proved essentially unsuccessful. More reliable experiments are reported by Kendall [4], who succeeded in confirming the theory in application to two-dimensional second-mode disturbances and three-dimensional (oblique) waves at a Mach number M = 4.5 and a Reynolds number $\mathrm{Re} = \sqrt{U_\infty x/\nu_\infty} = 1550$. The causes of the failures in studies of two-dimensional first-mode disturbances have yet to be explained.

Experimental studies of the stability of a supersonic boundary layer have been carried out at the Institute of Theoretical and Applied Mechanics of the Siberian Branch of the Academy of Sciences of the USSR [5]. Reliable data were obtained with the use of controllable artificial disturbances. They fully corroborate the basic principles of the theory of the stability of both plane-parallel [1] and slightly nonparallel [6] compressible flows. It has been established [7] that the wave number spectrum contains several maxima at a given frequency. The principal maximum corresponds to the results of the linear theory. The others could not be explained within the scope of the existing theory. The upstream incursion of disturbances has been observed in later experiments [8], but has not been investigated theoretically. Moreover, the spatial growth rates of waves whose fronts propagate at an angle $\chi < 45°$ relative to the main flow differ from those predicted by the theory of plane-parallel flows. In the present study, therefore, we continue the theoretical investigation of the growth of disturbances in a supersonic boundary layer, taking the new experimental data into account.

1. The stability of a supersonic boundary layer on a flat plate is analyzed both in the parallel-flow approximation and with allowance for departures from parallelism. The truncated Dunn–Lin equations (see [9]) are used in the first case, and the theory of [10] is used in the second case. In the calculations it is assumed that M = 4.0, $\mathrm{Re} = \sqrt{U_\infty x/\nu_\infty} = 600$, the Prandtl number Pr = 0.72, and the adiabatic exponent $\gamma = 1.4$. The viscosity–temperature relation is described by Sutherland's formula. Here U_∞ and ν_∞ are the velocity and viscosity at the outer edge of the boundary layer.

The disturbance is assumed to be a function of the dimensionless coordinates and time in the form

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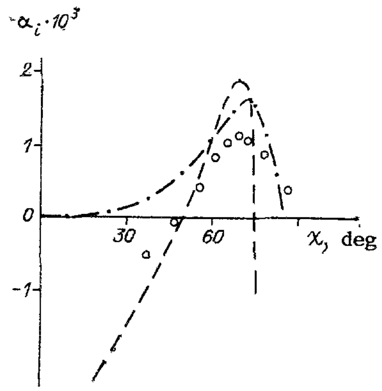

Fig. 1

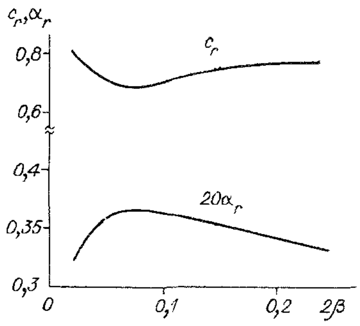

Fig. 2

$$Q_k(\bar{x}, \bar{y}, \bar{z}, \bar{t}) = A(\varepsilon\bar{x})\, q_k(\varepsilon\bar{x}, \bar{y}) \exp\left[i\left(\int\limits_{\bar{x}_0}^{\bar{x}} \alpha^0(\xi)\, d\xi + \beta\bar{z} - \omega\bar{t}\right)\right],$$

where q_k is the eigenfunction of the stability theory of locally parallel flows, which depends parametrically on $\bar{x}$; ε is a small parameter characterizing the departure of the flow from parallel; α^0 and β are wave numbers (which are complex in general); and ω is the angular frequency. The dimensionless coordinates are normalized to $\delta = \sqrt{x\nu_\infty/U_\infty}$, and the dimensionless time is normalized to $\tau = \delta/U_\infty$, the results of the stability analysis of the supersonic boundary layer are given for oscillations of the mass flow, whose spatial growth rate is determined from the relation $\alpha_r + i\alpha_i = (\delta/im)(\partial m/\partial x)$. The complex wave number α does not depend on y for plane-parallel flow, but does depend on it for nonparallel flow. The results given below for $-\alpha_i$ are obtained at the maximum amplitude of the mass flow inside the boundary layer.

Figure 1 shows the variation of the growth rate $-\alpha_i$ as a function of the angle of inclination of the wave vector relative to the direction of the main flow; this angle is defined as $\chi = \tan^{-1}(\beta_r/\alpha_r)$. Here $-\alpha_i$ depends on the constraint imposed on α and β. The principle used in the calculations is the same as the one underlying the processing of the experimental data, namely that β is a real quantity. The dashed curve in Fig. 1 corresponds to the experimental data [8], the dot-dashed line represents the theoretical results obtained in the parallel-flow approximation, and the circle-dots represent the results of the theory of slightly nonparallel flow at M = 4, Re = 600, and $\omega = 2\pi f\nu_\infty/U_\infty^2 = 0.213 \cdot 10^{-4}$. We see that the influence of nonparallelism on the growth rate at the given value of Re is strong and more pronounced than established in [10, 11] for Re = 1550 (M = 4.5) and Re = 780 (M = 4.0), respectively. We thus have further confirmation of the general theoretical conclusion that the influence of nonparallelism increases as Re decreases. A comparison of the data (Fig. 1) shows that the theoretical values of the spatial growth rate agree with the experimental at $\chi < 70°$. The greatest disparity is observed at large angles χ. Specifically, even at $\chi = 80°$ the disturbances decay in the $\bar{x}$-direction ($-\alpha_i < 0$). Special calculations show that the growth rate remains positive with increasing angle χ for $\chi > 80°$.

Inasmuch as the wave number α_r depends weakly on β (Fig. 2), the increase in the angle of inclination of the wave front is associated with an increase in β. Consequently, the wavelength, defined as $2\pi(\alpha_r^2 + \beta^2)^{-1/2}$, is shorter by roughly one tenth at (e.g.) $\chi = 84°$ than at $\chi = 0$. Under the conditions of the given experiment the dimensional wavelength is approximately equal to 3 mm at $\chi = 84°$ and is approximately equal to 5 mm at $\chi = 80°$.

The discrepancy between the theoretical and experimental data at large angles χ can be attributed to several causes. For large β the wavelength becomes comparable with the dimensions of the transducer, and the accuracy of the experimental results decreases. The initial spectrum of the disturbance in the experiments contains a small fraction of oscillations with large values of β. Their evolution can be influenced by the effects of nonlinear interaction of large-amplitude disturbances with small values of β.

Finally, nonparallelism affects not only the growth rate of an isolated wave with a fixed value of β, but also causes interaction to take place between waves with different values of α. In particular, two modes with identical values of β are observed in the parallel-flow approximation. Only the results of investigations of the first mode are shown in

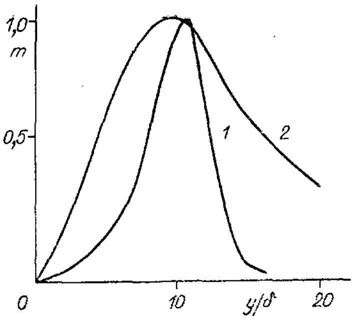

Fig. 3

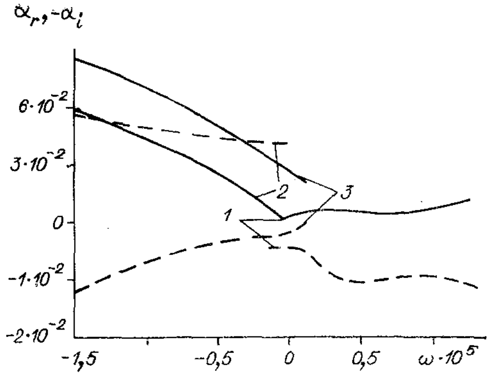

Fig. 4

Fig. 1. The second mode corresponds to disturbances that decay in the x-direction. As an example, we consider two modes with $2\beta_{1,2} = 0.347$. We infer from the theory of parallel flows that $2\alpha_1 = 0.032 - i0.7 \cdot 10^{-4}$ and $2\alpha_2 = 0.033 + i0.036$. The distribution of the mass-flow amplitude in the boundary layer for these modes is shown in Fig. 3. It is noteworthy that the true values of α_1 and α_2 are close to one another, even though their corresponding eigenfunctions differ. Allowance for nonparallelism shows that the rates of growth or decay of the peak of the mass-flow disturbance are $-\alpha_{i1} = 0.45 \cdot 10^{-3}$ and $\alpha_{i2} = -0.021$. It is evident from these data that the departure from parallelism exerts a strong influence for large β (particularly in the case of the second disturbance mode), and the results obtained without regard for interaction of the two modes becomes unreliable. The growth of disturbances with large β therefore remains an open question. It is important to note this fact in light of the reported observation [8] of the growth of such perturbations, even though their amplitudes were considered too small for them to be analyzed.

Figure 2 shows the wave number α_r and the "phase velocity" c_r corresponding to the x-direction as a function of the wave number β in the z-direction at the mass-flow peak.

2. It has been noted [7] that peaks are observed in the spectrum of longitudinal wave numbers α_r at a fixed frequency ω. This fact indicates that the disturbance as a function of time and the coordinate $\bar{x}$ has the form $\phi(\bar{x}, \bar{t}) = A \exp[i(\alpha_r \bar{x} - \omega \bar{t})]\Pi(\bar{x})$, where $\Pi(\bar{x})$ is a periodic function with period $2\pi/\alpha_0$. In this case the spectral representation is discrete with wave numbers $\alpha = \alpha_r \pm k\alpha_0$. This kind of spectrum can exist when a transient disturbance $\exp[i(\alpha_r \bar{x} - \omega \bar{t})]$, of which Tollmien–Schlichting waves are typical, interacts nonlinearly with steady-state waves $\exp(i\alpha_0 \bar{x})$. If we abide by the theory of slightly nonlinear interaction, the waveform of the steady-state disturbance $\phi_0(\bar{y}) \exp(i\alpha_0 \bar{x})$ is described by linear stability equations in the first approximation. Nonlinear interaction causes α_0 and the amplitude growth rate to vary slightly. An investigation at subsonic velocities shows that disturbances which decay in the linear approximation can grow as a result of nonlinear mode interaction. Finally, the upstream incursion of transient disturbances has been observed [8].

We have therefore investigated secondary wave modes in addition to the disturbances discussed in Sec. 1. Figure 4 shows the real and imaginary parts of the wave number α as functions of the frequency parameter at $\beta = 0$ and Re = 670 for three disturbance modes (the solid curves correspond to α_r, and the dashed curves to $-\alpha_i$). In the upper part of the boundary layer the constant-phase lines for the first and third modes in the xy plane correspond to the waves incident on the boundary layer, and those for the second mode correspond to radiating waves. The classification of incident and radiating waves is given in [9]. The first and third modes must therefore be excluded in the stability problem for parallel flows. However, they must be taken into account in the investigation of the growth of disturbances in nonparallel flow, in the presence of interaction, and in the vicinity of a localized source. The second mode corresponds to disturbances propagating upstream with strong attenuation. It is interesting to note the weak dependence of its dimensionless upstream decay rate on the frequency parameter. Additional calculations show that if Re is decreased from 670 to 200 at $\omega = 0.215 \cdot 10^{-4}$, the quantity $-\alpha_i$ increases only by 25%. However, its dimensional value increases considerably, because the thickness of the boundary layer decreases by about 0.4 in this case. The dimensionless real part of the wave number (absolute value) decreases by about 0.4, and its dimensional value remains constant.

It is evident from the results in Fig. 4 that steady-state waves corresponding to the first and third modes can exist in the boundary layer. As for the second mode, it undergoes upstream attenuation by relaxation in the limit $\omega \to 0$.

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FEATURES OF SEPARATED SUPERSONIC FLOW PULSATIONS AHEAD OF A SPIKE-TIPPED CYLINDER

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Use of a spike on a blunt body to decrease the aerodynamic resistance is limited by the adverse effect of intense flow pulsations, which depend on the shape of the front end of the body, the length of the spike, and the Mach number of the flow. Supersonic pulsation flow around such configurations have been investigated [1-7]. The intensive pulsations are excited by autooscillations in the forward separation zone ahead of the end with the spike.

The autooscillation system has distributed parameters; modeling it mathematically is a complex problem. The trend to construct simplified models makes it necessary to distinguish the most important components of the system: the oscillation system itself, the element which controls the input of energy into the system, feedback, and the energy source (the high-velocity flow) which surrounds the separation zone [8]. Delineating the basic elements of the autooscillation system requires detailed investigation, not only of the spectral and correlation characteristics of the pulsations, but also the separate stages of the flow pattern.

Experimental data on the magnitude and spectral composition of pressure pulsations on a cylinder with a conical spike, which has a 20° half-angle, are presented in [7], along with a preliminary discussion of the pulsation mechanism. Here we refine the description of the pulsation mechanism on a cylinder with a spike. We also give experimental data on

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